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THEOREM. *The locus of the vertices of all pyramids that have a proper coplanar quadrangle with three existing diagonals as a common base and which admit of square plane sections consists of three circles whose centers lie in, and whose planes are orthogonal each for each to, the three diagonals. Each of these circles is the intersection of two spheres with the two segments, like EF and GH, on each diagonal as diameters. If S_1 is a point on any of those circles attached to one of the diagonals, then any plane parallel to the plane determined by S_1 and this diagonal cuts the pyramid $S_1 \cdot ABCD$ in a square.*

3. For the sake of clearness the figure shows the construction for one diagonal, q', only. Any corresponding lines of the quadrangle and the square, like AB and $A'B'$ prolonged, meet in a point of s, the intersection of the planes R and P. The construction is still valid for the intersections R_1 and R_2 of X with the plane R. For these vertices the faces of the pyramid coincide with R. The reason for the validity in these limiting cases can easily be explained but, for the sake of brevity, will be omitted. In the language of central projection or perspective P may be considered as the plane of the object, the square $A'B'C'D'$; R as the picture-plane, S_1 as the center, s as the axis, q' as the vanishing-line of the perspective. E and F are the vanishing points of the two pairs of parallel sides of the square, and the quadrangle $ABCD$ is the perspective of the square.

4. If through S_1 we pass a plane parallel to R, it will cut P in a line t' whose perspective in a projective sense is the infinite line t of R.

By means of this perspective we can easily derive all the polar-properties of a conic from those of a circle. For instance, if we circumscribe a circle K' to the square and construct the pole T' of t' with respect to K', then T' will be projected into the pole T of the infinite line t of R with respect to the perspective K of K', which is a conic circumscribed to the quadrangle $ABCD$. By definition T is now the center of the conic K. According to the disposition in the figure this conic is an ellipse. To avoid the overloading of the figure this part of the construction, which would be simple enough, has been omitted. It follows also without difficulty that two conjugate polars of the circle through T' project into two conjugate diameters of the conic.

Thus we have reached the path by which in the historic development of projective geometry the theory of conics was derived from that of the circle.

BOOK REVIEWS.

SEND ALL COMMUNICATIONS TO W. H. BUSSEY, University of Minnesota.

The Elements of Non-Euclidean Plane Geometry and Trigonometry. By H. S. CARSLAW. Longmans, Green and Co., London, 1916. xii + 179 pages. $1.50.

In this work Professor Carslaw has given an excellent introduction to the classical non-euclidean geometries in the plane, an introduction of sufficient interest, ingenuity, and clearness to prove useful to a larger group of readers than that of "teachers of elementary geometry" for whom it was professedly written. The author has imposed on himself various limitations—not to depart from the plane; to avoid imaginary quantities, and, except in one chapter, coördinate systems; to build up a geometry independent of the principle of continuity; to avail himself of dividers (Streckenabträger) instead of compasses, wherever he could. We cannot expect such limitations to be an unmixed good for such a work. For instance, the introduction of imaginaries would have made the essential oneness of elliptic and hyperbolic geometry more apparent. There may be a question, too, as to whether it was worth while, in Chapter III, to avoid Dedekind's postulate so arduously, when Chapter IV had to use it repeatedly. This is, of course, only to say that there is no single all-surpassing way of approach to this subject.

The first chapter begins with a commentary on those axioms of Euclid, expressed or assumed, which are inconsistent with or unnecessary to the geometry to be developed. As Professor Carslaw does not set down his own fundamental assumptions, the reader takes them to be the corrected ones of Euclid. There follow proofs of a few theorems and problems common to non-euclidean and euclidean geometries.

The remainder of the first chapter, and all of the second, are historical. They tell briefly, but with a high degree of discrimination and clearness, of the work of the chief forerunners and founders of the new geometry—Saccheri, Legendre, Gauss, Schweikart, Bolyai, Lobatschevsky, Riemann.

Chapter III gives the theory of hyperbolic plane geometry, basing it on Hilbert's axiom of the existence of two parallels. First come theorems on parallels, angle of parallelism, the Saccheri (bi-rectangular isosceles) quadrilateral. From Lobatschevsky's remarkable correspondence between the five undetermined elements of a right triangle, and those of a tri-rectangular quadrilateral, Carslaw deduces a very elegant series of five associated right triangles. Points at infinity, ideal points (*i. e.*, those on and outside of the absolute) are defined as pencils of lines. The fundamental problems of construction—construction of parallel lines and of the distance corresponding to a given parallel angle—are next solved. Here for the first time compasses must replace dividers; but this does not imply a lapse into continuity, since Hilbert's axiom assures us that the circle and line whose intersection solves the problem do have a common point. The chapter ends with good discussions of the limiting curve (horocycle), the equidistant curve, and areas of triangles and polygons.

The reader who has been so incautious as to omit the preface will not expect continuity to appear unannounced in the fourth chapter (on trigonometry). Yet, on turning to p. 95, he will read: "Since we can find p to satisfy the equation $\Pi(p) = \pi/4$, there is a point Q on the Limiting Curve through P, such that the tangent at Q is parallel to the axis through P, in the opposite sense to that in which the axis is drawn." This means that the given horocycle and the equidistant curve whose distance from the axis satisfies the above equation will intersect; and this, barring accidents, requires that the two curves be continuous. The first open avowal of continuity comes on p. 106, where we read "Further, as a increases from 0 to $\infty, f(\alpha)$ diminishes continuously from $\pi/2$ to 0." Surely Carslaw should have adopted Dedekind's postulate explicitly at the beginning of the chapter. Aside from this, the development of trigonometry, according to Liebmann, is clearly shown.

Chapter V is devoted to the discussion of three coördinate systems (cartesian, limiting curve, and polar coördinates), and to computation of elements of arc and of area in all of them. One might call all three types "polar," since all have as coördinate curves concentric circles and their radii.

Two chapters are devoted to elliptic geometry and trigonometry, the treatment being that developed by Gérard, Mansion, and Coolidge.

The last chapter, on the consistency of the non-euclidean geometries and the impossibility of proving the parallel postulate," will be the most valuable one to those who study these geometries for the sake of better understanding the logical foundation of all geometry, particularly the euclidean. The consistency of the newer systems is shown by Poincaré's incisive method of representation on the euclidean plane. Carslaw names the euclidean circles which correspond to non-euclidean lines "nominal lines," the logarithm of the cross ratio of two variable and two fixed points, the "nominal length" of the "nominal segment" between the variable points etc.—a new terminology which has the great merit of avoiding the ambiguity of the word "ideal" previously used. The book closes with quotations from Poincaré and Hadamard and remarks of the author on the "truth" of non-euclidean geometry—the latter pointing out concisely the essential difference between "geometry, the pure science" and "geometry, the applied science"; between geometry as a branch of mathematics, and as a branch of physics.

The publishers and printers have done their part well. The reviewer has his own prejudices against thick paper, and thick lines and letters in diagrams, but recognizes them to be prejudices.

A few minor faults are as follows:

Page 8. *Note*. . . . "The point C must lie between B and the pole of AB." In the elliptic plane, between any line and its pole lie all other points.

Page 46, § 25. Two lines parallel to a third are themselves parallel only if they pass through the *same* end of the third line.

Page 51. In § 28 E has two different meanings.

Page 127. The first sentence of § 74 is decidedly obscure—it seems to speak of intersecting parallels.

The following misprints were noted.

Page 55, fig. 33, for h (upper point) read h'; page 63, line 5, for III′ read 3; page 124, line 9, for $\int_0^a \int_0^{2\pi}$ read $\int_0^{2\pi} \int_0^a$; page 148, line 13, for $\frac{sc''}{Mn}$ read $\frac{sc''}{MN}$; page 149, line 12, for ABC read DBC; page 156, line 3, for 29 read 92.

Professor Carslaw has given us, probably, the best balanced elementary book on non-euclidean geometry, both its history and its theory, which has been written; for that he deserves our sincere gratitude. For the future, let us hope that, aided by further coöperation of men so instinct with life and enthusiasm as Liebmann, he can continue to enrich the mathematical world.

EDWARD S. ALLEN.

THE UNIVERSITY OF MICHIGAN.

A History of Elementary Mathematics, with Hints on Methods of Teaching. By FLORIAN CAJORI, Ph.D. Revised and Enlarged Edition. The Macmillan Company, New York and London, 1917. viii + 324 pages. $1.75.

A general demand for information in regard to the historical development of all the great sciences, coupled with the questions that have recently been raised as to the advantage of omitting a considerable part of the present teaching of mathematics, makes Professor Cajori's *History of Elementary Mathematics* a timely and welcome addition to the libraries of both teachers and students. In accordance with the title, the emphasis of the book is upon the foundations of the science of mathematics. Some advanced theories are mentioned, but puzzling details are avoided, much to the relief of the general reader, who knows that many of the upper regions in mathematics can be reached only by the climbing of stairways which are invisible until one has acquired some Aladdin's lamp of knowledge.

The historical proof of the development of mathematics as a firm and progressive science, rather than as a changing mental plaything or business calculating machine, is suggested at once in the table of contents. This short and philosophical summary furnishes to the teacher of mathematics, or to the general reader of history, the material from antiquity to modern times which enables him to place the subject where it belongs—among the old and reliable and constantly developing sciences. At practically every point which is taken up, quite full and satisfactory references are given, so that authorities on past and present historical material are placed before the reader. Those who are familiar with Professor Cajori's *History of Mathematics*, dated December, 1893, will recognize with pleasure the reappearance of many of the well-preserved mathematical antiques which he then used to exhibit clearly and forcibly the contributions of the science during the ages. The authorship and the construction of many of these antiques are again shown to be insolvable problems, but the discussion of the possible originators forms an interesting and valuable part of the new book.

Among racial contributions the book presents with great appreciation the work of the Hindus and the Arabs, and mentions the recently discovered numeral records of the Mayas of Central America as an early attempt on the part of